SELF-DUAL GAUGE FIELDS ON M5-BRANES

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[Nambu 73] [Takhtajan 94]

NAMBU POISSON BRACKET

- Nambu Poisson bracket

$$\{f,g,h\} = P^{\dot{\mu}\dot{\nu}\dot{\lambda}}(\partial_{\dot{\mu}}f)(\partial_{\dot{\nu}}g)(\partial_{\dot{\lambda}}h)$$

$$\dot{\mu},\dot{\nu},\dot{\lambda} = \dot{1},\dot{2},\dot{3}$$

- Skew-symmetry

$$\{f,g,h\} = -\{g,f,h\} = -\{h,g,f\}$$

- Leibniz rule

$$\{fg,h_1,h_2\} = -f\{g,h_1,h_2\} = -\{f,h_1,h_2\}g$$

- Jacobi identity

$$\{f,g,\{h_1,h_2,h_3\}\} =$$

$$\{\{f,g,h_1\},h_2,h_3\} + \{h_1,\{f,g,h_2\},h_3\} + \{h_1,h_2,\{f,g,h_3\}\}$$

VOLUME-PRESERVING-DIFFEOMORPHISM (VPD)

For a 3D space, the coordinate transf. $\delta y^{\dot{\mu}} = \kappa^{\dot{\mu}}$

preserves the volume-form $dy^{\dot{1}} dy^{\dot{2}} dy^{\dot{3}}$

if $\partial_{\dot{\mu}} \kappa^{\dot{\mu}} = 0$ so that $\kappa^{\dot{\lambda}} = \epsilon^{\dot{\lambda}\dot{\mu}\dot{\nu}} \partial_{\dot{\mu}} \Lambda_{\dot{\nu}}$

Transfs. on scalars can be expressed via NP bracket as

$$\delta \Phi = \kappa^{\dot{\mu}} \partial_{\dot{\mu}} \Phi = \sum_a \{f_a, g_a, \Phi\}$$

GAUGE SYMMETRY

- In free field limit, the gauge potential is

$$\delta_\Lambda b^{\dot{\mu}} = \kappa^{\dot{\mu}}$$

- Abelian 2-form gauge potential can be defined as

$$\delta_\Lambda b_{\dot{\mu}\dot{\nu}} = \partial_{\dot{\mu}}\Lambda_{\dot{\nu}} - \partial_{\dot{\nu}}\Lambda_{\dot{\mu}}$$
$$\delta_\Lambda b_{\mu\dot{\nu}} = \partial_{\mu}\Lambda_{\dot{\nu}} - \partial_{\dot{\nu}}\Lambda_{\mu}$$
$$\mu = 0, 1, 2$$

- The field strength is

$$H_{\dot{\lambda}\dot{\mu}\dot{\nu}} = \partial_{\dot{\lambda}} b_{\dot{\mu}\dot{\nu}} + \partial_{\dot{\mu}} b_{\dot{\nu}\dot{\lambda}} + \partial_{\dot{\nu}} b_{\dot{\lambda}\dot{\mu}}$$
$$H_{\lambda\dot{\mu}\dot{\nu}} = \partial_{\lambda} b_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b_{\lambda\dot{\nu}} + \partial_{\dot{\nu}} b_{\lambda\dot{\mu}}$$

GAUGE SYMMETRY (DEFORMED)

- gauge transformation (VPD)

[Ho, Matsuo 08]
[Ho, Imamura, Matsuo, Shiba 08]

$$B_\mu{}^{\dot\mu} \equiv \epsilon^{\dot\mu\dot\nu\dot\rho}\partial_{\dot\nu} b_{\mu\dot\rho},$$

$$\begin{aligned}
\delta_\Lambda \Phi &= g\kappa^{\dot\rho}\partial_{\dot\rho}\Phi \qquad (\Phi = X^i, \Psi) \\
\delta_\Lambda b^{\dot\mu} &= \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu}, \\
\delta_\Lambda B_\mu{}^{\dot\mu} &= \partial_\mu \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} B_\mu{}^{\dot\mu} - g(\partial_{\dot\nu}\kappa^{\dot\mu})B_\mu{}^{\dot\nu}
\end{aligned}$$

- Field strengths

$$\begin{aligned}
\mathcal{H}_{\lambda\dot\mu\dot\nu} &= \epsilon_{\dot\mu\dot\nu\dot\lambda}\mathcal{D}_\lambda X^{\dot\lambda} \\
&= H_{\lambda\dot\mu\dot\nu} - g\epsilon^{\dot\sigma\dot\tau\dot\rho}(\partial_{\dot\sigma} b_{\lambda\dot\tau})\partial_{\dot\rho} b_{\dot\mu\dot\nu}, \\
\mathcal{H}_{\dot1\dot2\dot3} &= g^2\{X^{\dot1}, X^{\dot2}, X^{\dot3}\} - \frac{1}{g} \\
&= H_{\dot1\dot2\dot3} + \frac{g}{2}(\partial_{\dot\mu} b^{\dot\mu}\partial_{\dot\nu} b^{\dot\nu} - \partial_{\dot\mu} b^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu}) + g^2\{b^{\dot1}, b^{\dot2}, b^{\dot3}\}
\end{aligned}$$

SEIBERG-WITTEN MAP FOR VPD

Exact results: [Chen, Furuuchi, Ho, Takimi 10]

$$\theta^{ijk}(t) = \theta^{ijk}\frac{1}{1-\frac{t}{6}(\theta^{a_1a_2a_3}H_{a_1a_2a_3})}$$

$$\rho(x^i) = x^i + \hat{b}^i = e^{\partial_t + \frac{1}{2}\theta^{ijk}(t)b_{ij}\partial_k}\, x^i\Big|_{t=0}$$

$$A \equiv \partial_t + \frac{1}{2}\theta^{ijk}(t)b_{ij}\partial_k, \quad B \equiv \frac{1}{2}\theta^{ijk}(t)(\partial_i\Lambda_j - \partial_j\Lambda_i)\partial_k$$

$$\hat{\varphi} = e^{A}\varphi\Big|_{t=0}$$

$$e^{A+B}e^{-A} - 1 = \hat{\kappa}^i(t)\partial_i + \mathcal{O}(\Lambda^2)$$

M5 IN C (3+3)

[Ho, Matsuo 08]
[Ho, Imamura, Matsuo, Shiba 08]

$$S = S_X + S_\Psi + S_{gauge} \qquad S_{gauge} = S_{\mathcal{H}^2} + S_{CS}$$

$$\begin{aligned}
S_X &= \int d^3x d^3y \left[-\frac{1}{2}(\mathcal{D}_\mu X^i)^2 - \frac{1}{2}(\mathcal{D}_{\dot\lambda} X^i)^2 \right. \\
&\quad \left. -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot\mu}, X^i, X^j\}^2 - \frac{g^4}{12}\{X^i, X^j, X^k\}^2 \right] \\
S_\Psi &= \int d^3x d^3y \left[\frac{i}{2}\overline{\Psi}\Gamma^\mu \mathcal{D}_\mu \Psi + \frac{i}{2}\overline{\Psi}\Gamma^{\dot\rho}\mathcal{D}_{\dot\rho}\Psi \right. \\
&\quad \left. +\frac{ig^2}{2}\overline{\Psi}\Gamma_{\dot\mu i}\{X^{\dot\mu}, X^i, \Psi\} - \frac{ig^2}{4}\overline{\Psi}\Gamma_{ij}\Gamma_{\dot1\dot2\dot3}\{X^i, X^j, \Psi\} \right] \\
S_{\mathcal{H}^2} &= \int d^3x d^3y \left[-\frac{1}{12}\mathcal{H}^2_{\dot\mu\dot\nu\dot\rho} - \frac{1}{4}\mathcal{H}^2_{\lambda\dot\mu\dot\nu} \right] \\
S_{CS} &= \int d^3x d^3y \, \epsilon^{\mu\nu\lambda}\epsilon^{\dot\mu\dot\nu\dot\lambda} \left[-\frac{1}{2}\partial_{\dot\mu} b_{\mu\dot\nu}\partial_\nu b_{\lambda\dot\lambda} + \frac{g}{6}\partial_{\dot\mu} b_{\nu\dot\nu}\epsilon^{\dot\rho\dot\sigma\dot\tau}\partial_{\dot\sigma} b_{\lambda\dot\rho}(\partial_{\dot\lambda} b_{\mu\dot\tau} - \partial_{\dot\tau} b_{\mu\dot\lambda}) \right]
\end{aligned}$$

- covariant derivatives

$$\begin{aligned}
\mathcal{D}_\mu \Phi &= \partial_\mu \Phi - g B_\mu{}^{\dot\mu}\partial_{\dot\mu}\Phi. \\
\mathcal{D}_{\dot\mu}\Phi &= \frac{g^2}{2}\epsilon_{\dot\mu\dot\nu\dot\rho}\{X^{\dot\nu}, X^{\dot\rho}, \Phi\}
\end{aligned}$$

SUPERSYMMETRY

$$
\begin{aligned}
\delta_\epsilon X^i &= i\bar{\epsilon}\Gamma^i\Psi \\
\delta_\epsilon \Psi &= \mathcal{D}_\mu X^i \Gamma^\mu \Gamma^i \epsilon + \mathcal{D}_{\dot{\mu}} X^i \Gamma^{\dot{\mu}} \Gamma^i \epsilon \\
&\quad -\frac{1}{2}\mathcal{H}_{\mu\dot{\nu}\dot{\rho}}\Gamma^\mu\Gamma^{\dot{\nu}\dot{\rho}}\epsilon - \mathcal{H}_{\dot{1}\dot{2}\dot{3}}\Gamma_{\dot{1}\dot{2}\dot{3}}\epsilon \\
&\quad -\frac{g^2}{2}\{X^{\dot{\mu}}, X^i, X^j\}\Gamma^{\dot{\mu}}\Gamma^{ij}\epsilon + \frac{g^2}{6}\{X^i, X^j, X^k\}\Gamma^{ijk}\Gamma^{\dot{1}\dot{2}\dot{3}}\epsilon \\
\delta_\epsilon b_{\dot{\mu}\dot{\nu}} &= -i(\bar{\epsilon}\Gamma_{\dot{\mu}\dot{\nu}}\Psi) \\
\delta_\epsilon b_{\mu\dot{\nu}} &= -i\left(1 + g\mathcal{H}_{\dot{1}\dot{2}\dot{3}}\right)\bar{\epsilon}\Gamma_\mu\Gamma_{\dot{\nu}}\Psi + ig(\bar{\epsilon}\Gamma_\mu\Gamma_i\Gamma_{\dot{1}\dot{2}\dot{3}}\Psi)\partial_{\dot{\nu}}X^i
\end{aligned}
$$

NP M5 TO NC D4

[Ho, Imamura, Matsuo, Shiba 08]

- Double Dimensional Reduction

$$y^{\dot{3}} \sim y^{\dot{3}} + 2\pi R$$

- VPD becomes Area-Preserving-Diff.

$$b_{\dot{1}\dot{2}} = b^{\dot{3}} = 0$$
$$b_{\dot{\alpha}\dot{3}} = a_{\dot{\alpha}}$$
$$b_{\mu\dot{3}} = a_{\mu}$$
$$b_{\mu\dot{\alpha}} \quad \text{integrated out}$$

$$\kappa^{\dot{\mu}} = \epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}}\partial_{\dot{\nu}}\Lambda_{\dot{\lambda}}$$
$$\Lambda_{\dot{1}}, \Lambda_{\dot{2}}$$
$$\Lambda_{\dot{3}} = \lambda$$

- Seiberg-Witten limit for NC D4 reinterpreted for NP M5

$$\epsilon \to 0$$

[Chen, Furuuchi, Ho, Takimi 10]

$$\ell_P \sim \epsilon^{1/3}, \qquad g_{\mu\nu} \sim 1, \qquad g_{\dot{\mu}\dot{\nu}} \sim \epsilon, \qquad C_{\dot{\mu}\dot{\nu}\dot{\lambda}} \sim 1$$

- open F1 on D-brane in B field => Poisson bracket
- open M2 on M5 in C field => Nambu-Poisson bracket.
- generalization to Dp on D(p+2) in constant RR (p+1)-form background

[Ho, Yeh 11]

NP M5 TO NP D4

- Double Dimensional Reduction

$$x^2 \sim x^2 + 2\pi R$$

- VPD survives, U(1) appears

$$b^{\dot{\mu}}$$

$$b_{2\dot{\mu}} = a_{\dot{\mu}}$$

$$b_{\alpha\dot{\mu}} \quad \xrightarrow{dual} \quad a_{\alpha}$$

We will focus on the gauge fields.

- Low energy limit for NP M5 reinterpreted for NP D4

$$\ell_s \sim \epsilon^{1/2}, \qquad g_s \sim \epsilon^{-1/2}, \qquad g_{\alpha\beta} \sim 1, \qquad g_{\dot{\mu}\dot{\nu}} \sim \epsilon, \qquad C_{\dot{\mu}\dot{\nu}\dot{\lambda}} \sim 1$$

$$g_s \ell_s \ll 1$$

radius much smaller then 1/E

Background fields:

$$C_{012} = \frac{1}{(2\pi)^2 \ell_P^3} C_{\dot{1}\dot{2}\dot{3}}$$

No NP structure in (012) directions.

$$2\pi\alpha' B_{01} = 2\pi \ell_s^2 R C_{012} = \frac{C_{\dot{1}\dot{2}\dot{3}}}{2\pi}$$

finite B-field background (no NC)

GAUGE SYMMETRY

- gauge transformation

$$\delta b^{\dot\mu} = \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu}b^{\dot\mu}$$

$$\delta a_A = \partial_A\lambda + g(\kappa^{\dot\nu}\partial_{\dot\nu}a_A + a_{\dot\nu}\partial_A\kappa^{\dot\nu})$$

$A = (\mu \ \text{ or } \ \dot\mu) = 0, 1, \dot1, \dot2, \dot3$ $\qquad \lambda \equiv \Lambda_2$

- field strengths

$$\mathcal{H}_{\dot1\dot2\dot3} = \partial_{\dot\mu}b^{\dot\mu} + \frac{1}{2}g(\partial_{\dot\nu}b^{\dot\nu}\partial_{\dot\rho}b^{\dot\rho} - \partial_{\dot\nu}b^{\dot\rho}\partial_{\dot\rho}b^{\dot\nu}) + g^2\{b^{\dot1}, b^{\dot2}, b^{\dot3}\}$$

$$\mathcal{F}_{\dot\mu\dot\nu} \equiv \mathcal{H}_{\dot\mu\dot\nu2} = F_{\dot\mu\dot\nu} + g[\partial_{\dot\sigma}b^{\dot\sigma}F_{\dot\mu\dot\nu} - \partial_{\dot\mu}b^{\dot\sigma}F_{\dot\sigma\dot\nu} - \partial_{\dot\nu}b^{\dot\sigma}F_{\dot\mu\dot\sigma}]$$

$$\mathcal{F}_{\alpha\dot\mu} = {V^{-1}}_{\dot\mu}{}^{\dot\nu}(F_{\alpha\dot\nu} + gF_{\dot\nu\dot\delta}\hat{B}_\alpha{}^{\dot\delta})$$

$$\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} + g[-F_{\alpha\dot\mu}\hat{B}_\beta{}^{\dot\mu} - F_{\dot\mu\beta}\hat{B}_\alpha{}^{\dot\mu} + gF_{\dot\mu\dot\nu}\hat{B}_\alpha{}^{\dot\mu}\hat{B}_\beta{}^{\dot\nu}]$$

$$V_{\dot\nu}{}^{\dot\mu} \equiv \delta^{\dot\mu}_{\dot\nu} + g\partial_{\dot\nu} b^{\dot\mu}$$

$$M^{\alpha\beta}_{\dot\mu\dot\nu} \equiv V_{\dot\mu\dot\rho} V_{\dot\nu}{}^{\dot\rho} \delta^{\alpha\beta} - g\epsilon^{\alpha\beta} F_{\dot\mu\dot\nu}$$

$$(M^{-1})^{\dot\mu\dot\lambda}_{\alpha\gamma} M^{\gamma\beta}_{\dot\lambda\dot\nu} = \delta^{\dot\mu}_{\dot\nu}\delta^{\beta}_{\alpha}$$

$$\hat{B}_{\alpha}{}^{\dot\mu} \equiv (M^{-1})^{\dot\mu\dot\nu}_{\alpha\beta}(V_{\dot\nu}{}^{\dot\lambda}\partial^{\beta} b_{\dot\lambda} + \epsilon^{\beta\gamma} F_{\gamma\dot\nu})$$

ACTION

[Ho, Yeh 11]

$$S_{gauge} = \int d^2x d^3y \left\{ -\frac{1}{2}\mathcal{H}_{\dot{1}\dot{2}\dot{3}}\mathcal{H}^{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}\mathcal{F}_{\beta\dot{\mu}}\mathcal{F}^{\beta\dot{\mu}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} \right\}$$

- To the lowest order

$$S_{gauge} \simeq \int d^2x d^3y \left\{ -\frac{1}{2}(H_{\dot{1}\dot{2}\dot{3}} + F_{01})^2 - \frac{1}{4}F_{AB}F^{AB} \right\}$$

- One vector field, two symmetries

GENERALIZATION TO DP

[Ho, Yeh 11]

- Nambu-Poisson bracket with (p-1) slots

$$\{f_1, f_2, \cdots, f_{p-1}\} \equiv \epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}}\partial_{\dot{\mu}_1}f_1\partial_{\dot{\mu}_2}f_2\cdots\partial_{\dot{\mu}_{p-1}}f_{p-1}$$

- Gauge fields

$$b^{\dot{\mu}_1} = \frac{1}{(p-2)!}\epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}}b_{\dot{\mu}_2\cdots\dot{\mu}_{p-2}}$$

$$X^{\dot{\mu}} = \frac{y^{\dot{\mu}}}{g} + b^{\dot{\mu}}$$

$$\delta a_A = [D_A, \lambda] + g(\kappa^{\dot{\mu}}\partial_{\dot{\mu}}a_A + a_{\dot{\mu}}\partial_A\kappa^{\dot{\mu}})$$

$$A = 0, 1, 2, \cdots, p$$

$$\begin{aligned}
\mathcal{F}_{\dot{\mu}\dot{\nu}} &= F_{\dot{\mu}\dot{\nu}} + g[\partial_{\dot{\sigma}}b^{\dot{\sigma}}F_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}}b^{\dot{\sigma}}F_{\dot{\sigma}\dot{\nu}} - \partial_{\dot{\nu}}b^{\dot{\sigma}}F_{\dot{\mu}\dot{\sigma}}] \\
\mathcal{F}_{\alpha\dot{\mu}} &= {V^{-1}}_{\dot{\mu}}^{\ \dot{\nu}}(F_{\alpha\dot{\nu}} + gF_{\dot{\nu}\dot{\delta}}\hat{B}_{\alpha}^{\ \dot{\delta}}) \\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot{\mu}}\hat{B}_{\beta}^{\ \dot{\mu}} - F_{\dot{\mu}\beta}\hat{B}_{\alpha}^{\ \dot{\mu}} + gF_{\dot{\mu}\dot{\nu}}\hat{B}_{\alpha}^{\ \dot{\mu}}\hat{B}_{\beta}^{\ \dot{\nu}}]
\end{aligned}$$

$$\mathcal{H}_{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}} \equiv g^{p-2}\{X^{\dot{\mu}_1}, X^{\dot{\mu}_1}, \cdots, X^{\dot{\mu}_{p-1}}\} - \frac{1}{g} = \partial_{\dot{\mu}}b^{\dot{\mu}} + \mathcal{O}(g)$$

$$\delta \mathcal{F}_{AB} = [\mathcal{F}_{AB}, \lambda - g\kappa^{\dot{\mu}}\partial_{\dot{\mu}}]$$

- symmetry algebra $[\delta_1, \delta_2] = \delta_3$

$$\begin{aligned} \lambda_3 &= [\lambda_1, \lambda_2] + g(\kappa_2^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_1 - \kappa_1^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_2) \\ \kappa_3^{\dot{\mu}} &= g(\kappa_2^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_1^{\dot{\mu}} - \kappa_1^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_2^{\dot{\mu}}) \end{aligned}$$

- action $a_A = a_A^{U(1)} + a_A^{SU(N)}$

$$\begin{aligned} S_{gauge}^{Dp} &= \int d^2x d^{p-1}y \left\{ -\frac{1}{2}\frac{1}{(p-1)!}\mathcal{H}_{\dot{\mu}_1\cdots\dot{\mu}_{p-1}}\mathcal{H}^{\dot{\mu}_1\cdots\dot{\mu}_{p-1}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta}^{U(1)} \right. \\ & \left. -\frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}^{U(1)}\mathcal{F}_{U(1)}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}\mathcal{F}_{\beta\dot{\mu}}^{U(1)}\mathcal{F}_{U(1)}^{\beta\dot{\mu}} - \frac{1}{4}\mathrm{tr}\left(\mathcal{F}_{AB}^{SU(N)}\mathcal{F}_{SU(N)}^{AB}\right)\right\} \end{aligned}$$

NON-ABELIAN SELF-DUAL TWO-FORM GAUGE THEORY FOR MULTIPLE M5-BRANES

[Ho, Huang, Matsuo 11]

- Two criteria for a theory of *N* M5:

N M5 on S^1 with $R \to 0$
≈ *N* D4
≈ *SU(N)* Super Yang-Mills theory

N well separated M5 on S^1 with finite *R*
≈ *N* copies of single M5 (on S^1 with finite *R*)
≈ *N* copies of 6D Abelian chiral gauge theory

GAUGE ALGEBRA

- The idea: $D_i \equiv \partial_i + gB_{i5}^{(0)}.$

[Ho, Huang, Matsuo 11]

$$\begin{aligned}
\delta B_{i5} &= [D_i, \Lambda_5] - \partial_5 \Lambda_i + g[B_{i5}^{(\mathrm{KK})}, \Lambda_5^{(0)}], \\
\delta B_{ij} &= [D_i, \Lambda_j] - [D_j, \Lambda_i] + g[B_{ij}, \Lambda_5^{(0)}] - g[F_{ij}, \partial_5^{-1} \Lambda_5^{(\mathrm{KK})}],
\end{aligned}$$

$$F_{ij} \equiv g^{-1}[D_i, D_j] = \partial_i B_{j5}^{(0)} - \partial_j B_{i5}^{(0)} + g[B_{i5}^{(0)}, B_{j5}^{(0)}].$$

- more explicitly

$$\begin{aligned}
\delta B_{i5}^{(0)} &= [D_i, \Lambda_5^{(0)}], \\
\delta B_{i5}^{(\mathrm{KK})} &= [D_i, \Lambda_5^{(\mathrm{KK})}] - \partial_5 \Lambda_i^{(\mathrm{KK})} + g[B_{i5}^{(\mathrm{KK})}, \Lambda_5^{(0)}], \\
\delta B_{ij}^{(0)} &= [D_i, \Lambda_j^{(0)}] - [D_j, \Lambda_i^{(0)}] + g[B_{ij}^{(0)}, \Lambda_5^{(0)}], \\
\delta B_{ij}^{(\mathrm{KK})} &= [D_i, \Lambda_j^{(\mathrm{KK})}] - [D_j, \Lambda_i^{(\mathrm{KK})}] + g[B_{ij}^{(\mathrm{KK})}, \Lambda_5^{(0)}] - g[F_{ij}, \partial_5^{-1} \Lambda_5^{(\mathrm{KK})}].
\end{aligned}$$

- Redundancy in parametrizing gauge symmetry

$$\delta\Lambda_i^{(\mathrm{KK})} = [D_i, \lambda^{(\mathrm{KK})}], \qquad \delta\Lambda_5^{(\mathrm{KK})} = \partial_5 \lambda^{(\mathrm{KK})}.$$

- Field strengths

$$\begin{aligned}
H_{ij5}^{(0)} &\equiv F_{ij} \equiv g^{-1}[D_i, D_j], \\
H_{ij5}^{(\mathrm{KK})} &\equiv [D_i, B_{j5}^{(\mathrm{KK})}] - [D_j, B_{i5}^{(\mathrm{KK})}] + \partial_5 B_{ij}, \\
H_{ijk}^{(0)} &\equiv [D_i, B_{jk}^{(0)}] + [D_j, B_{ki}^{(0)}] + [D_k, B_{ij}^{(0)}], \\
H_{ijk}^{(\mathrm{KK})} &\equiv [D_i, B_{jk}^{(\mathrm{KK})}] + [D_j, B_{ki}^{(\mathrm{KK})}] + [D_k, B_{ij}^{(\mathrm{KK})}] \\
&\quad + g[F_{ij}, \partial_5^{-1} B_{k5}^{(\mathrm{KK})}] + g[F_{jk}, \partial_5^{-1} B_{i5}^{(\mathrm{KK})}] + g[F_{ki}, \partial_5^{-1} B_{j5}^{(\mathrm{KK})}].
\end{aligned}$$

- Jacobi identities

$$\begin{aligned}
\sum_{(3)} [D_i, H_{jk5}^{(0)}] &= 0, \\
\sum_{(3)} [D_i, H_{jk5}^{(\mathrm{KK})}] &= \partial_5 H_{ijk}^{(\mathrm{KK})}, \\
\sum_{(4)} [D_i, H_{jkl}^{(0)}] &= 0, \\
\sum_{(4)} [D_i, H_{jkl}^{(\mathrm{KK})}] &= g \sum_{(6)} [H_{ij5}^{(0)}, \partial_5^{-1} H_{kl5}^{(\mathrm{KK})}],
\end{aligned}$$

- Gauge transformation of field strengths

$$
\begin{aligned}
\delta H_{ij5}^{(0)} &= g[H_{ij5}^{(0)}, \Lambda_5^{(0)}], \\
\delta H_{ij5}^{(\text{KK})} &= g[H_{ij5}^{(\text{KK})}, \Lambda_5^{(0)}], \\
\delta H_{ijk}^{(0)} &= g[H_{ijk}^{(0)}, \Lambda_5^{(0)}] + g[H_{ij5}^{(0)}, \Lambda_k^{(0)}] + g[H_{jk5}^{(0)}, \Lambda_i^{(0)}] + g[H_{ki5}^{(0)}, \Lambda_j^{(0)}], \\
\delta H_{ijk}^{(\text{KK})} &= g[H_{ijk}^{(\text{KK})}, \Lambda_5^{(0)}].
\end{aligned}
$$

- Gauge symmetry algebra

$$[\delta, \delta'] = \delta'',$$

$$
\begin{aligned}
\Lambda_5''^{(0)} &= g[\Lambda_5^{(0)}, \Lambda_5'^{(0)}], \\
\Lambda_5''^{(\text{KK})} &= g[\Lambda_5^{(0)}, \Lambda_5'^{(\text{KK})}] - g[\Lambda_5'^{(0)}, \Lambda_5^{(\text{KK})}], \\
\Lambda_i'' &= g[\Lambda_5^{(0)}, \Lambda_i'] - g[\Lambda_5'^{(0)}, \Lambda_i].
\end{aligned}
$$

- The action [Ho, Huang, Matsuo 11]

$$S^{(0)} = \frac{2\pi R}{4} T_{M5} T_{M2}^{-2} \int d^5x \, \mathrm{Tr}(F_{ij}F^{ij}), \qquad A_i = 2\pi R B_{i5}^{(0)}.$$

$$S^{(\mathrm{KK})} = \frac{1}{4} T_{M5} T_{M2}^{-2} \int d^6x \, \mathrm{Tr}\left(\frac{1}{6}\epsilon^{ijklm} H_{ijk}^{(\mathrm{KK})} \left[H_{lm5}^{(\mathrm{KK})} + \frac{1}{6}\epsilon_{lmnpq} H^{(\mathrm{KK})npq}\right]\right).$$

- Equation of motion

$$\epsilon^{ijklm}\partial_k \left(H_{lm5}^{(\mathrm{KK})} + \frac{1}{6}\epsilon_{lmnpq} H^{(\mathrm{KK})npq}\right) = 0.$$

$$\epsilon^{ijklm} \left(H_{lm5}^{(\mathrm{KK})} + \frac{1}{6}\epsilon_{lmnpq} H^{(\mathrm{KK})npq}\right) = \epsilon^{ijklm}\Phi_{lm}^{(\mathrm{KK})}, \qquad \epsilon^{ijklm}[D_k, \Phi_{lm}^{(\mathrm{KK})}] = 0.$$

- gauge symmetry

$$B_{lm}^{(\mathrm{KK})} \to B_{lm}^{\prime(\mathrm{KK})} \equiv B_{lm}^{(\mathrm{KK})} + \partial_5^{-1}\Phi_{lm}^{(\mathrm{KK})},$$

OTHER PROPOSALS

[Chong-Sun's proposals 11]
[Douglas 11; Lambert, Papageorgakis 11]
[Samtleben, Sezgin, Wimmer 11]
[Others (non-Abelian gerbes etc.)]

Issues:
Extra gauge potential
No single M5 limit or no D4 limit
No nontrivial 5^{th} momentum distribution among fields

- Our model
- nonlocal only in 5th direction
- has both D4 limit and single M5 limit
- feature: KK modes interact only indirectly through 0-mode, later verified by Yu-tin's work

CONCLUSION

- M5 in large C
- Dp in RR (p+1)-form background
- Multiple M5 theory (gauge field sector)